ARithmetische Geometrie OberSeminar

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Prismatic cohomology

Prismatic cohomology has recently been defined in [1]. For any *prism* (A, I), where in particular A is a ring with Frobenius lift ϕ and $I \subset A$ is an invertible ideal, and scheme X over A/I, it defines a cohomology theory taking values in A-modules equipped with a Frobenius. Its fibre over A/I is Hodge(-Tate) cohomology, while other fibres are related to crystalline, de Rham, and étale cohomology. It simplifies, unifies, and expands much of the recent work [2], [3], [4].

Talks

1. Talk: δ-rings

Define δ-rings and establish basic results concerning the category of δ-rings. Identify the free and cofree δ-rings. Show that the category of p-complete perfect δ-rings is equivalent to the category of perfect rings. ([1, Section 1])

2. Talk: Prisms

Define distinguished elements in δ-rings, and prisms. Discuss various equivalent characterizations on the condition that $I \subset A$ defines a prism. Show that perfect prisms are equivalent to perfectoid rings. Define the prismatic site. ([1, Section 2])

3. Talk: Crystalline comparison

In characteristic p, identify certain δ-rings with divided power envelopes. Use this to compare prismatic cohomology and crystalline cohomology. ([1, Section 3])

4. Talk: Hodge-Tate comparison

Prove the comparison between the specialization of prismatic cohomology along $A \to A/I$ and differential forms, by reduction to characteristic p. ([1, Section 3])

5. Talk: de Rham comparison

Prove the comparison between the specialization of prismatic cohomology along $A \xrightarrow{\phi} A \to A/I$ and de Rham cohomology. ([1, Section 3])

6. Talk: André's lemma

André's lemma asserts that many natural maps of perfectoid rings are flat (modulo p). Its original proof gave this result only in the almost world, and relied on adic spaces. Use prismatic cohomology to give a direct algebraic proof, and use the lemma to show that all Zariski closed immersions of perfectoid spaces are in fact strongly Zariski closed (contradicting the counterexample claimed in [5, Section II.2]). ([1, Section 4])

7. Talk: Étale comparison

Prove that the (derived) ϕ-invariants in the base extension of prismatic cohomology along $A \to A[I^{-1}]/p^n$ agree with étale cohomology of the generic fibre of X with $\mathbb{Z}/p^n\mathbb{Z}$-coefficients. ([1, Section 4])

8. Talk: Examples

In the Breuil-Kisin and Breuil-Kisin-Fargues examples, apply prismatic cohomology to recover the main results of [2] and [3]. Moreover, construct q-de Rham cohomology, verifying conjectures made in [4].

9. Talk: Comparison with [2]

Prove that the A_{inf}-cohomology of [2] is given by prismatic cohomology over the prism $(A_{\mathrm{inf}}, \ker\tilde{\theta})$, via realizing prismatic cohomology in terms of q-de Rham complexes to obtain a strictly functorial map towards the A_{inf}-cohomology.

10. Talk: Comparison with [3]

Prove that the Breuil-Kisin cohomology of [3] is given by prismatic cohomology over the prism $(\mathfrak{S}, (E))$. Prove that for quasiregular semiperfectoid R, the ring $\pi_0\mathrm{TP}(R;\mathbb{Z}_p)$ equipped with its cyclotomic Frobenius is a δ-ring.

References

[1] B. Bhatt, P. Scholze, *Prisms and prismatic cohomology*, in preparation.

[2] B. Bhatt, M. Morrow, P. Scholze, *Integral p-adic Hodge theory*, arXiv:1602.03148

[3] B. Bhatt, M. Morrow, P. Scholze, *Topological Hochschild homology and integral p-adic Hodge theory*, arXiv:1802.03261

[4] P. Scholze, *Canonical q-deformations in arithmetic geometry*, Ann. Fac. Sci. Toulouse Math. (6) 26 (2017), no. 5, 1163–1192.

[5] P. Scholze, *On torsion in the cohomology of locally symmetric varieties*, Ann. of Math. (2) 182 (2015), no. 3, 945–1066.